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ASER of Rectangular QAM For *L*–Branch EGC Receiver With Phase Estimation Error Over Nakagami-M Fading Channels

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Abstract

Quadrature amplitude modulation (QAM) is the most commonly used digital modulation scheme, which is used in almost all modern communications like 4G and 5G wireless communication standards. QAM contains a combination of two PAMs, which can be viewed as a combined amplitude/phase modulation or as a complex amplitude-modulated carrier [1]. QAM is a bandwidth profitable modulation class which is used in the domain of multimedia transmission. The profits of bandwidth efficiency and high power make the application of QAM techniques in LTE [2]. QAM, is an encouraging way of enhancing the bandwidth efficiency, among other modulation techniques. Since the constellation size of the QAM signaling can be adjusted

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based on the channel quality, the QAM schemes are known as adaptive modulation schemes[3-5]. There are various QAM techniques that are termed as square QAM (SQAM), rectangular QAM (RQAM) and cross QAM (XQAM). RQAM is a generic modulation scheme since it includes many prominent modulations such as orthogonal binary frequency-shift keying, quadrature phase-shift keying, binary phase-shift keying and SQAM [5]. RQAM has practical implementation in the field of microwave communications, telephone line modems and high rate wireless communications [6], [7].

The equal gain combining (EGC) diversity technique can be used to exterminate the detrimental effects of fading. In the EGC receiver due to noise in the phase recovery loop and interference, the estimated phase is imperfect causing phase estimation error [8]. The performance analysis of the RQAM scheme for various fading environments have been described in many studies. In [9], the ASER analysis of RQAM is expressed for cooperative diversity systems, under Rayleigh fading channels. The ASER expression is shown for the RQAM technique under Nakagami-m fading conditions in [10][11]. ASER expression of the RQAM is developed for MRC receiver over i.i.d. Nakagami-m fading conditions in [12]. TheASER expression for an L-branch EGC receiver with RQAM scheme over Nakagami-m fading with phase estimation error is reported in this article.

Among different diversity combining techniques, EGC can achieve a performance close to the optimum performance of the MRC with relatively less implementation complexity [13]. To achieve the best performance from a diversity combiner, it is required to accurately estimate the instantaneous amplitude and phase of transmitted signal. In an EGC combiner, received faded copies of the transmitted signal are equiphased and each is weighted by unity gain [1]. Equiphasing operation requires the phases to be estimated. Due to the presence of Doppler spread in carrier frequency, thermal noise and other unwanted signals in the phase estimation circuit, the estimated phase may not be accurate. The error introduced in this estimation is known as the phase estimation error. To have effective mobile communication, there is a need to design a receiver which can operate satisfactorily in the presence of phase estimation error. In this work, the PDF-based approach is applied to obtain the expression of ASER for an RQAM with two approximations.

2. PDF of EGC Receiver Output SNR

The instantaneous output SNR, for an L-branch EGC receiver dealing with carrier phase error, can be shown as

$$\gamma = \left(\sum_{l=1}^{L} \sqrt{\gamma_l} \cos\left(\varphi_l\right) \right)^2 , \qquad (1)$$

where, $\varphi_l = \phi_l - \hat{\phi}_l$ is the phase estimation error. The RV ϕ_l is the carrier phase and $\hat{\phi}_l$ is the estimated carrier phase by a first-order PLL. The PDF of Nakagami-m distributed RV γ_l is given as

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$$f_{\gamma_l}\left(\gamma_l\right)=\frac{m^m\gamma_l^{m-1}}{\Gamma\left(m\right)\left(\bar{\gamma}\right)^m}e^{-\left(\frac{m}{\bar{\gamma}}\gamma_l\right)} \quad . \qquad (2)$$

Where, m is the fading parameter and $\bar{\gamma}$ is the average SNR. Using RV transformation, the PDF of EGC receiver output SNR with phase error can be noted as [14]

$$f_\gamma\left(\gamma\right)=\left(\frac{m}{\bar{\gamma}}\right)^{L\left(\frac{m}{2}-\frac{1}{4}\right)}\frac{\Gamma^L\left(m-\frac{1}{2}\right)\zeta^{L\left(\frac{m}{2}+\frac{1}{4}\right)}}{2^{L\left(\frac{m}{2}-\frac{3}{2}\right)}2(\Gamma\left(m\right))^L}$$

$$\times\frac{\left(\sqrt{\gamma}\right)^{L\left(m-\frac{1}{2}\right)-2}}{\Gamma\left(L\left(m-\frac{1}{2}\right)\right)}e^{-\sqrt{\frac{2m\zeta}{\bar{\gamma}}}\left(\sqrt{\gamma}\right)} \quad , \qquad (3)$$

where ζ represents the loop SNR and $\Gamma(.)$ is the Gamma function.

3. Average Symbol Error Rate Analysis

The ASER for different modulation schemes is generally determined as [5]

$$P_e=\int_0^\infty P_e\left(e/\gamma\right)f_\gamma\left(\gamma\right)d\gamma \quad . \qquad (4)$$

Where $P_e(e/\gamma)$ is the conditional error probability for AWGN and it is specified for M-ary RQAM as [15]

$$P_e(e/\gamma) = 2VQ\left(u\sqrt{\gamma}\right) + 2WQ\left(v\sqrt{\gamma}\right)$$

$$-4VWQ\left(u\sqrt{\gamma}\right)Q\left(v\sqrt{\gamma}\right), \qquad (5)$$

Where, $V = 1 - \left(\frac{1}{M_I}\right)$; $W = 1 - \left(\frac{1}{M_Q}\right)$; $\beta = \frac{d_Q}{d_I}$;

$M = M_I \times M_Q$; Q(.) is Gaussian Q-function; and

$u = \sqrt{\frac{6}{\left(M_I^2 - 1\right) + \left(M_Q^2 - 1\right)\beta^2}}$; $v = \beta u$; d_Q and d_I are the quadrature phase and in-phase decision distance, respectively. Employing two different approximations of Gaussian Q -function, the ASER is calculated.

3.1. ASER Analysis Using Chernoff Approximation of Gaussian Q -Function

We can use the Chernoff approximation for $Q(.)$ function, i.e., $Q(\tau) \approx \frac{1}{2}e^{-\frac{\tau^2}{2}}$ for RQAM [16][17]. Therefore (5) can be rewritten as [15]

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$$P_e\left(e/\gamma\right)=V\times e^{\frac{-u^2\gamma}{2}}+W\times e^{\frac{-v^2\gamma}{2}}-VW\times e^{\frac{-(u^2+v^2)\gamma}{2}}.$$

(6)

Substituting (3) and (6) in (4), the ASER for RQAM changes into

$$P_{e,chern}=\left(\frac{\left(\frac{m}{\bar{\gamma}}\right)^{L\left(\frac{m}{2}-\frac{1}{4}\right)}\Gamma^L\left(m-\frac{1}{2}\right)\zeta^{L\left(\frac{m}{2}+\frac{1}{4}\right)}}{2^{L\left(\frac{m}{2}-\frac{3}{2}\right)}2(\Gamma\left(m\right))^L\Gamma\left(L\left(m-\frac{1}{2}\right)\right)}\right)$$

$$\times\left[V\int_0^\infty\left(\gamma\right)^{L\left(\frac{m}{2}-\frac{1}{4}\right)-1}e^{-\left(\frac{u^2\gamma}{2}+\sqrt{\frac{2m\zeta\gamma}{\bar{\gamma}}}\right)}d\gamma\right.$$

$$+W\int_0^\infty\left(\gamma\right)^{L\left(\frac{m}{2}-\frac{1}{4}\right)-1}e^{-\left(\frac{v^2\gamma}{2}+\sqrt{\frac{2m\zeta\gamma}{\bar{\gamma}}}\right)}d\gamma$$

$$\left.-VW\int_0^\infty\left(\gamma\right)^{L\left(\frac{m}{2}-\frac{1}{4}\right)-1}e^{-\left(\frac{\left(u^2+v^2\right)\gamma}{2}+\sqrt{\frac{2m\zeta\gamma}{\bar{\gamma}}}\right)}d\gamma\right].\quad(7)$$

Applying [18, (3.462.1)] and after mathematical manipulation in (7), the ASER expression becomes

$$P_{e,chern} = \left(\left(\frac{m}{\bar{\gamma}} \right)^{L\left(\frac{m}{2}-\frac{1}{4}\right)} \frac{\Gamma^L\left(m-\frac{1}{2}\right)}{2^{L\left(\frac{m}{2}-\frac{3}{2}\right)} \{\Gamma(m)\}^L} \zeta^{L\left(\frac{m}{2}+\frac{1}{4}\right)} \right)$$

$$\times\left[V(u)^{-\left\{L\left(m-\frac{1}{2}\right)\right\}} \exp\left(\frac{m\zeta}{2\bar{\gamma}u^2}\right) D_{-\left\{L\left(m-\frac{1}{2}\right)\right\}}\left(\frac{1}{u}\sqrt{\frac{2m\zeta}{\bar{\gamma}}}\right)\right.$$

$$+W(v)^{-\left\{L\left(m-\frac{1}{2}\right)\right\}} \exp\left(\frac{m\zeta}{2\bar{\gamma}v^2}\right) D_{-\left\{L\left(m-\frac{1}{2}\right)\right\}}\left(\frac{1}{v}\sqrt{\frac{2m\zeta}{\bar{\gamma}}}\right)$$

$$-\left\{ VW\left(u^2+v^2\right)^{-\frac{L\left(m-\frac{1}{2}\right)}{2}} \exp\left(\frac{m\zeta}{2\bar{\gamma}(u^2+v^2)}\right)\right.$$

$$\left.\left.\times D_{-\left\{L\left(m-\frac{1}{2}\right)\right\}}\left(\sqrt{\frac{2m\zeta}{\bar{\gamma}(u^2+v^2)}}\right)\right\}\right], \quad (8)$$

where, $D_{-\varepsilon}(.)$ is the parabolic cylinder function.

3.2. ASER Analysis Employing Chiani Approximation in Gaussian Q-Function

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Using the approximation [19], $erfc(\varpi) \simeq \frac{e^{-\varpi^2}}{6} + \frac{e^{-\frac{4\varpi^2}{3}}}{2}$ in $Q(.)$ function, the expression of (5) can be revised as [16]

$$P_e(e/\gamma) = 2V\left[\frac{1}{12}e^{\frac{-u^2\gamma}{2}} + \frac{1}{4}e^{\frac{-2u^2\gamma}{3}}\right]$$

$$+2W\left[\frac{1}{12}e^{\frac{-v^2\gamma}{2}} + \frac{1}{4}e^{\frac{-2v^2\gamma}{3}}\right]$$

$$-4VW\left[\frac{1}{144}e^{-\left(\frac{u^2+v^2}{2}\right)\gamma} + \frac{1}{48}e^{-\left(\frac{u^2}{2}+\frac{2v^2}{3}\right)\gamma}\right.$$

$$\left.+\frac{1}{48}e^{-\left(\frac{2u^2}{3}+\frac{v^2}{2}\right)\gamma} + \frac{1}{16}e^{-\left(\frac{2u^2+2v^2}{3}\right)\gamma}\right]. \quad (9)$$

Inserting, the expressions of (3) and (9) into (4), the ASER can be shown as

$$P_{e,chiani} = \Lambda_1 + \Lambda_2 - \Lambda_3, \quad (10)$$

where

$$\Lambda_1 = \left(\frac{V\left(\frac{m}{\bar{\gamma}}\right)^{L\left(\frac{m}{2}-\frac{1}{4}\right)} \Gamma^L\left(m-\frac{1}{2}\right)\zeta^{L\left(\frac{m}{2}+\frac{1}{4}\right)}}{2^{L\left(\frac{m}{2}-\frac{3}{2}\right)}\left\{\Gamma(m)\right\}^L \Gamma\left(L\left(m-\frac{1}{2}\right)\right)} \right)$$

$$\times\int_0^\infty \left[(\gamma)^{L\left(\frac{m}{2}-\frac{1}{4}\right)-1} \frac{1}{12} e^{\frac{-u^2\gamma}{2}-\sqrt{\frac{2m\zeta}{\bar{\gamma}}}\left(\sqrt{\gamma}\right)} \right.$$

$$\left. +(\gamma)^{L\left(\frac{m}{2}-\frac{1}{4}\right)-1} \frac{1}{4} e^{\frac{-2u^2\gamma}{3}-\sqrt{\frac{2m\zeta}{\bar{\gamma}}}\left(\sqrt{\gamma}\right)} \right] d\gamma \quad , \qquad (11)$$

$$\Lambda_2 = \left(\frac{W\left(\frac{m}{\bar{\gamma}}\right)^{L\left(\frac{m}{2}-\frac{1}{4}\right)} \Gamma^L\left(m-\frac{1}{2}\right)\zeta^{L\left(\frac{m}{2}+\frac{1}{4}\right)}}{2^{L\left(\frac{m}{2}-\frac{3}{2}\right)}\left\{\Gamma(m)\right\}^L \Gamma\left(L\left(m-\frac{1}{2}\right)\right)} \right)$$

$$\times\int_0^\infty \left[(\gamma)^{L\left(\frac{m}{2}-\frac{1}{4}\right)-1} \times \frac{1}{12} e^{\frac{-v^2\gamma}{2}-\sqrt{\frac{2m\zeta}{\bar{\gamma}}}\left(\sqrt{\gamma}\right)} \right.$$

$$\left. +(\gamma)^{L\left(\frac{m}{2}-\frac{1}{4}\right)-1} \times \frac{1}{4} e^{\frac{-2v^2\gamma}{3}-\sqrt{\frac{2m\zeta}{\bar{\gamma}}}\left(\sqrt{\gamma}\right)} \right] d\gamma \quad , \qquad (12)$$

and similarly,

$$\Lambda_3 = \left(\frac{2VW\left(\frac{m}{\bar{\gamma}}\right)^{L\left(\frac{m}{2}-\frac{1}{4}\right)} \Gamma^L\left(m-\frac{1}{2}\right)\zeta^{L\left(\frac{m}{2}+\frac{1}{4}\right)}}{2^{L\left(\frac{m}{2}-\frac{3}{2}\right)}\{\Gamma(m)\}^L\,\Gamma\left(L\left(m-\frac{1}{2}\right)\right)} \right)$$

$$\times\int_0^\infty \left[(\gamma)^{L\left(\frac{m}{2}-\frac{1}{4}\right)-1}\frac{1}{144}e^{-\left(\frac{u^2+v^2}{2}\right)\gamma-\sqrt{\frac{2m\zeta}{\bar{\gamma}}}\left(\sqrt{\gamma}\right)}\right.$$

$$+(\gamma)^{L\left(\frac{m}{2}-\frac{1}{4}\right)-1}\frac{1}{48}e^{-\left(\frac{u^2}{2}+\frac{2v^2}{3}\right)\gamma-\sqrt{\frac{2m\zeta}{\bar{\gamma}}}\left(\sqrt{\gamma}\right)}$$

$$+(\gamma)^{L\left(\frac{m}{2}-\frac{1}{4}\right)-1}\frac{1}{48}e^{-\left(\frac{2u^2}{3}+\frac{v^2}{2}\right)\gamma-\sqrt{\frac{2m\zeta}{\bar{\gamma}}}\left(\sqrt{\gamma}\right)}$$

$$\left.+(\gamma)^{L\left(\frac{m}{2}-\frac{1}{4}\right)-1}\frac{1}{16}e^{-\left(\frac{2u^2+2v^2}{3}\right)\gamma-\sqrt{\frac{2m\zeta}{\bar{\gamma}}}\left(\sqrt{\gamma}\right)}\right]d\gamma \quad . \quad (13)$$

Applying [18, (3.462.1)], the expressions in (11), (12) and (13) can be obtained as

$$\Lambda_1 = \left(\left(\frac{m}{\bar{\gamma}}\right)^{L\left(\frac{m}{2}-\frac{1}{4}\right)} \frac{\Gamma^L\left(m-\frac{1}{2}\right)\zeta^{L\left(\frac{m}{2}+\frac{1}{4}\right)}}{2^{L\left(\frac{m}{2}-\frac{3}{2}\right)+1}\{\Gamma(m)\}^L} \right)$$

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$$\times\left[\frac{1}{3}V(u)^{-\left\{L\left(m-\frac{1}{2}\right)\right\}}\exp\left(\frac{m\zeta}{2\bar{\gamma}u^2}\right)D_{-\left\{L\left(m-\frac{1}{2}\right)\right\}}\left(\sqrt{\frac{2m\zeta}{\bar{\gamma}u^2}}\right)\right.$$

$$+\left\{V\left(\frac{4u^2}{3}\right)^{-\frac{\left\{L\left(m-\frac{1}{2}\right)\right\}}{2}}\exp\left(\frac{3m\zeta}{8\bar{\gamma}u^2}\right)\right.$$

$$\left.\left.\times D_{-\left\{L\left(m-\frac{1}{2}\right)\right\}}\left(\sqrt{\frac{3m\zeta}{2\bar{\gamma}u^2}}\right)\right\}\right], \qquad (14)$$

$$\Lambda_2=\left(\left(\frac{m}{\bar{\gamma}}\right)^{L\left(\frac{m}{2}-\frac{1}{4}\right)}\frac{\Gamma^L\left(m-\frac{1}{2}\right)\zeta^{L\left(\frac{m}{2}+\frac{1}{4}\right)}}{2^{L\left(\frac{m}{2}-\frac{3}{2}\right)+1}\left\{\Gamma(m)\right\}^L}\right)$$

$$\times\left[\frac{1}{3}W(v)^{-\left\{L\left(m-\frac{1}{2}\right)\right\}}\exp\left(\frac{m\zeta}{2\bar{\gamma}v^2}\right)D_{-\left\{L\left(m-\frac{1}{2}\right)\right\}}\left(\sqrt{\frac{2m\zeta}{\bar{\gamma}v^2}}\right)\right.$$

$$+W\left(\frac{4v^2}{3}\right)^{-\frac{\left\{L\left(m-\frac{1}{2}\right)\right\}}{2}}\exp\left(\frac{3m\zeta}{8\bar{\gamma}v^2}\right)$$

$$\left.\times D_{-\left\{L\left(m-\frac{1}{2}\right)\right\}}\left(\sqrt{\frac{3m\zeta}{2\bar{\gamma}v^2}}\right)\right]. \qquad (15)$$

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And,

$$\Lambda_3 = \left(\left(\frac{m}{\overline{\gamma}}\right)^{L\left(\frac{m}{2}-\frac{1}{4}\right)} \frac{\Gamma^L\left(m-\frac{1}{2}\right)\zeta^{L\left(\frac{m}{2}+\frac{1}{4}\right)}}{2^{L\left(\frac{m}{2}-\frac{3}{2}\right)+1}\left\{\Gamma(m)\right\}^L} \right)$$

$$\times\left[\frac{1}{18}VW\left(u^2+v^2\right)^{-\frac{\left\{L\left(m-\frac{1}{2}\right)\right\}}{2}} \exp\left(\frac{m\zeta}{2\overline{\gamma}\left(u^2+v^2\right)}\right)\right.$$

$$\times D_{-\left\{L\left(m-\frac{1}{2}\right)\right\}}\left(\sqrt{\frac{2m\zeta}{\overline{\gamma}\left(u^2+v^2\right)}}\right)$$

$$+\frac{1}{6}VW\left(\frac{3u^2+4v^2}{3}\right)^{-\frac{\left\{L\left(m-\frac{1}{2}\right)\right\}}{2}} \exp\left(\frac{3m\zeta}{2\overline{\gamma}\left(3u^2+4v^2\right)}\right)$$

$$\times D_{-\left\{L\left(m-\frac{1}{2}\right)\right\}}\left(\sqrt{\frac{6m\zeta}{\overline{\gamma}\left(3u^2+4v^2\right)}}\right)$$

$$+\frac{1}{6}VW\left(\frac{4u^2+3v^2}{3}\right)^{-\frac{\left\{L\left(m-\frac{1}{2}\right)\right\}}{2}} \exp\left(\frac{3m\zeta}{2\overline{\gamma}\left(4u^2+3v^2\right)}\right)$$

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$$\times D_{-\left\{L\left(m-\frac{1}{2}\right)\right\}}\left(\sqrt{\frac{6m\zeta}{\overline{\gamma}\left(4u^{2}+3v^{2}\right)}}\right)$$

$$+\frac{1}{2}VW\left\{\frac{4}{3}\left(u^{2}+v^{2}\right)\right\}^{-\frac{\left\{L\left(m-\frac{1}{2}\right)\right\}}{2}}\exp\left(\frac{3m\zeta}{8\overline{\gamma}\left(u^{2}+v^{2}\right)}\right)$$

$$\left.\times D_{-\left\{L\left(m-\frac{1}{2}\right)\right\}}\left(\sqrt{\frac{3m\zeta}{2\overline{\gamma}\left(u^{2}+v^{2}\right)}}\right)\right]. \qquad (16)$$

4. Numerical Results and Discussions

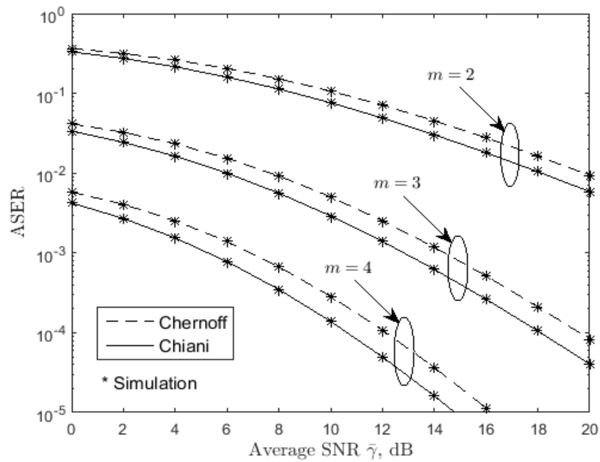

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Fig. 1. ASER vs. average SNR with $M_I = 4$, $M_Q = 2$, $L = 2$, $\zeta = 1$ and $\beta = 1$.

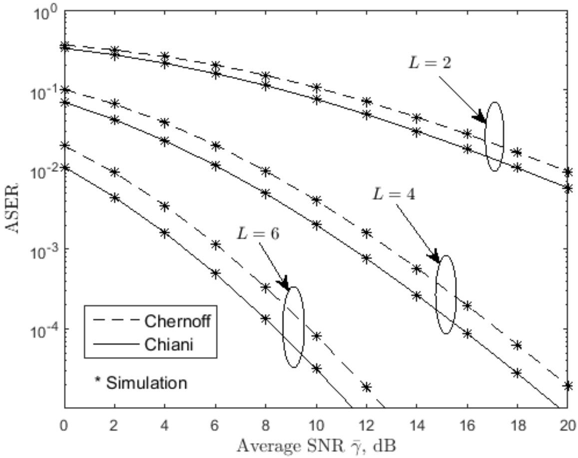

Fig. 2. ASER vs. average SNR with $M_I = 4$, $M_Q = 2$, $m = 2$, $\zeta = 1$ and $\beta = 1$.

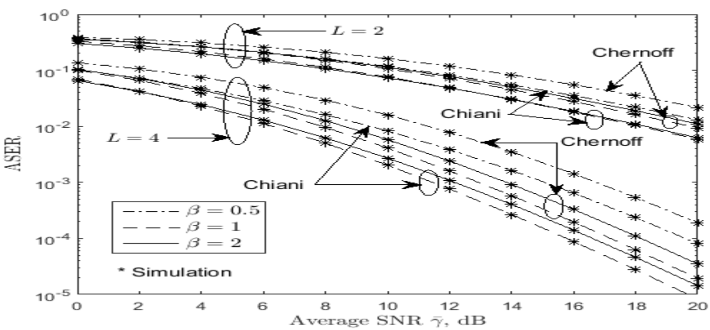

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Fig. 3. ASER vs. average SNR with $M_I = 4$, $M_Q = 2$, $m = 2$ and $\zeta = 1$.

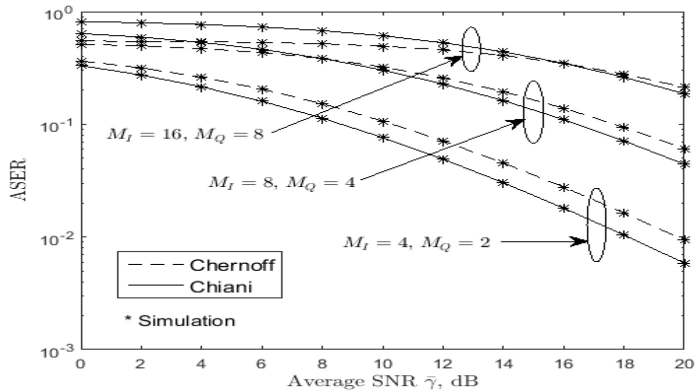

Fig. 4. ASER vs. average SNR with $L = 2$, $m = 2$, $\zeta = 1$ .and $\beta = 1$.

Table 1. Relative difference between accepted results and approximate values of ASER applying the Chernoff and Chiani approximations, for 4×2 RQAM with $m = 2, L = 2$ and $\beta = 1$

SNR (dB)	ASER		
	Accepted Result [20]	Approximation	Relative Difference (%)

0	0.5438916	Chernoff:	0.364308	33.01827055
		Chiani:	0.331637	39.02516604
5	0.3012875	Chernoff:	0.233208	22.59619135
		Chiani:	0.186543	38.08471974
10	0.0814489	Chernoff:	0.106765	31.08218773
		Chiani:	0.0762429	6.39173764

Numerical results of ASER for arbitrary branch EGC receiver with phase estimation error are given in the Figures for RQAM scheme. The results are displayed for different levels of fading parameter m , the diversity order L , and the decision distance ratio β . In Fig. 1, the ASER versus average SNR per branch is given for 4×2 RQAM ($M_I=4$ and $M_Q=2$) scheme with $L=2$, $\zeta=1$, $\beta=1$ and m is varied. It is observed from Fig. 1, that with the addition in the values of the fading index m of the channel, the ASER performance improves. Higher the value of fading parameter m , the channel becomes better. Table 1 is

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made for the absolute relative difference between accepted results and approximate values with the Chernoff and Chiani approximations of ASER for 4×2 RQAM with $m=2$ and $\beta=1$. The accepted values of ASER at low average SNR, are mentioned in [20] for 4×2 RQAM with selection combining diversity receiver over Nakagami- m fading channels. Both the accepted values of ASER and the ASER values with the approximations are for two ($L=2$) received antennas at the receiver. The ASER performance is better with the two approximations due to the consideration of EGC receiver and phase estimation error in the system.

In Fig. 2, the ASER performance versus Average SNR is plotted for 4×2 RQAM with defined values of $m=2$, $\zeta=1$, $\beta=1$ and varying amounts of the diversity order L. It is observed that with the raising in the diversity order L, the ASER performance of the receiver becomes better. This is because of the low probability of deep fades in all the diversity branches at the same instant, hence reduces the probability of outage and improves the system performance.

In Fig. 3, the ASER performance versus average SNR is plotted for 4×2 RQAM schemes by changing β and with $m=2$ **and** $\zeta=1$**.** From Fig. 3, it is seen that the ASER performance outperforms for $\beta = 1$ than the ASER performance with $\beta = 0.5$ and $\beta = 2$ for both $L = 2$ and $L = 4$. For $\beta = 1$, quadrature phase distance (d_Q) and in-phase distance (d_I) are equal; hence the minimum ASER can be obtained for the channel.

In Fig. 4, the ASER performance versus average SNR is plotted for different M -ary RQAM signals with $L = 2$, $m = 2$, $\zeta = 1$ and $\beta = 1$ in the system. It is noticed that the ASER performance of 4×2 ($M = 8$) RQAM improves, compared to 8×4 ($M = 32$) RQAM and 16×8 ($M = 128$) RQAM. The ASER performance degrades with the increment in the constellation size, i.e. the values of M . This is because, when the number of bits per transmitted symbol are increased, the probability of ASER rises.

It can be noticed from the Figures and Table 1 that the ASER of Chiani approximation outperforms the ASER for the Chernoff approximation. In Chiani approximation two exponential terms are used for Gaussian Q -function, so it provides preferably accurate ASER measure when compared to the Chernoff approximation which consists of one exponential term. Therefore, the ASER performance of RQAM over Nakagami- m fading channels realized applying the Chiani approximation points out more improvement. On the other hand, the computational complexity is more in Chiani approximation. Curves in the Figures have been compared with the available results in the literature and found that they are closely matching. Computer simulated results are also plotted in the Figures, to verify the correctness of the evaluated results. Consequently, as expected, the ASER performance enhances with an increase in diversity order L and fading parameter m . Similarly, the ASER performance falls with an increase in constellation size M and with Chernoff approximation.

Conclusions

We investigated the ASER performance of an L -branch EGC receiver with phase error under Nakagami- m fading channels. The new ASER expressions are derived using Chernoff and Chiani approximations with the RQAM technique. Closed-form formulations are derived for ASER with Gamma and parabolic cylinder functions, which are attainable in mathematical software. Derived ASER expressions are compared with the expressions available in literature and the results are validated with computer simulated results. The numerical results of the expressions and computer-simulated results are in close accord. The reaction of the fading parameter and the number of receiving antennas on the receiver performance is illustrated.

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